γ-ray wavelength as standard for atomic scales


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The lattice parameter $a$ of Si is the most accurate atomic dimensions length standard in use. In determining $a$ a relative uncertainty of $2.9 \times 10^{-5}$ has been reached [1]. The lattice constant, however, is sensitive to the ambient temperature and pressure, as well as to the chemical purity and perfection of the reference crystal. To reproduce its absolute value with a comparable precision all these parameters have to be controlled very accurately. An alternative easily reproducible length standard is always desirable.

γ-radiation which nuclei emit without recoil, the Mössbauer radiation, could be used as a length standard of atomic dimensions that is easily reproducible with an uniquely high accuracy. The wavelength $\lambda_M \simeq 8.6 \times 10^{-11}$ m of the $^{57}$Fe Mössbauer radiation at $E_M \simeq 14.4$ keV, for example is reproducible with an accuracy of at least $10^{-11}$ without any special precautions regarding temperature, pressure and chemical composition of the environment in which the radiating nuclei are placed. Mössbauer radiation is available from radioactive sources or it can be generated easily at synchrotron facilities. The energy $E_M$ and the wavelength $\lambda_M$, however, are known with an accuracy of $10^{-5}$ only [2].

The idea of the presented experiment is to measure the ratio of the Si lattice constant to the Mössbauer wavelength. A relative uncertainty of $1.9 \times 10^{-7}$ has been achieved. This allows us to determine the value of $\lambda_M$ with the same uncertainty by using the known value of $a$. The used experimental technique exploits the fact that Bragg’s law for exact backscattering $\lambda = 2d_{hkl}$ establishes a direct relation between the interplanar distance $d_{hkl} = a/\sqrt{h^2 + k^2 + l^2}$ of the reflecting atomic planes $(hkl)$ in the reference crystal and the wavelength $\lambda$ of the reflected radiation. This is used to calibrate the instrument for measuring wavelengths in units of $a$, the $\lambda$-instrument. By selecting suitable Miller indices $(hkl)$ the calibration can be performed in any region of interest.

The set-up is shown schematically in Figure 1. A special silicon crystal (Si) is used as reference, its lattice constant $a = 5.43102030(36) \times 10^{-10}$ m is known with a relative uncertainty of $8 \times 10^{-8}$ [3]. To reproduce this value the crystal is kept in an evacuated thermostat at 22.500 °C (295.650 K) with an absolute accuracy of 5 mK and a stability of 1 mK. The thermostat is mounted on a four circle goniometer allowing to orientate the crystal according to the desired back-reflections. The symmetry Bragg-reflection (7 7 7) is used. The wavelength of the reflected radiation is varied by changing its angle of incidence $\Psi$ to the (7 7 7) atomic planes. The diode separately detects the prompt pulse after the $\lambda$-instrument and the backscattered one after the 40 ns time of flight towards the crystal and back. If the wavelength coincidences with $\lambda_M$ it excites the $^{57}$Fe nuclei in the α-iron foil (F) installed downstream. The Mössbauer radiation is emitted with an average time delay of 141 ns, this allows to discriminate it from any other radiation. The detection of this delayed radiation signals that the $\lambda$-instrument is tuned to $\lambda_M$. 
Two experiments were performed at different synchrotron radiation facilities, one at the wiggler beamline BW4 at HASYLAB (DESY, Hamburg). The ratio \( \tilde{a} = a/\tilde{\lambda}_M \) at \( t = 22.500 \degree C \) was determined as \( \tilde{a} = 6.3132548(12) \). \( \tilde{\lambda}_M \) is the Mössbauer wavelength inside silicon as it differs from the vacuum value due to the index of refraction. The second experiment was carried out at the undulator beam line 3-ID at the Advanced Photon Source (ANL, Argonne). The accuracy was improved by a factor of seven because a higher count rate allowed a better angular collimation of the beam. The value of \( \tilde{a} \) at \( t = 22.500 \degree C \) was determined as:

\[
\tilde{a} = 6.3132548(12)
\]

This value agrees with the one determined from the HASYLAB measurements. By using the known value of \( a \) for the reference crystal and taking effects of refraction into account, we obtain:

\[
\lambda_M = 8.6025474(16) \times 10^{-11} \text{ m}
\]

for the wavelength of the \(^{57}\text{Fe} \) Mössbauer radiation and

\[
E_M = 14.412497(3) \text{ keV}
\]

for its energy \( E_M = hC/\lambda_M \).

The accuracy can be improved by at least an order of magnitude if the beam divergence is reduced to a sub-\( \mu \)rad level. The future x-ray free-electron laser sources with much better emittance will certainly allow this.

### References

